THE ELEMENTS OF *Plain Trigonometry* DEMONSTRATED: AND ITS Practice and Use EXPLAINED.

By *John Harris*, M. A. and F. R. S.

LONDON:
Printed by *E. Midwinter*, for *D. Midwinter* and *T. Leigh* at the *Rose and Crown* in St. *Paul's Church-yard*. MDCCIII.

MVSEVM
BRITAN
NICVM

696

TO THE READER.

THO' there are very many Books on this Subject already extant, some of which are very well done; yet I perswade my self, that the Plainness and the Brevity of this Tract will make it very useful and serviceable to many Persons to whom some larger Books may seem both Difficult and Tedious:

And I am apt to think that a few Hints may be met with here, which will serve to give the Reader a much clearer and distinct notion of the Nature of Trigonometry, *than is ordinarily to be met with in Books of this Subject; especially as to the* Rationale *of* Stating a Case, *and* Forming a Canon *to work: by which, till a Person can readily do, he is by no means a Master of this Science.*

For tho' a Man may only

by Rules contrived to help the Memory, retain a thing a great while, especially if Practice make it ready and familiar to him; yet 'tis, I think, much better, in order to prevent Mistakes or Blunders; or to avoid being puzled with some Difficulties, which either the Nature of a Question, or the Manner of proposing it, may sometimes occasion: 'Tis much better, I say, That the Learner be let into the whole Mystery, see

the Thing to the very bottom, and be permitted to take no Steps but where he hath ſolid Demonſtration to ſtand or proceed upon; and then he will be out of danger of either, being deceived himſelf, or expoſing the Reputation of his Teacher.

I queſtion not but this little Book will be an Inſtance of the Poſſibility and Facility of proceeding in ſuch a Method; and will convince any unprejudic'd Perſon, that

the

the Theory *and* Practice *together, of any part of* Mathematicks, *may be taught as ſoon as the Practice alone without the Theory.*

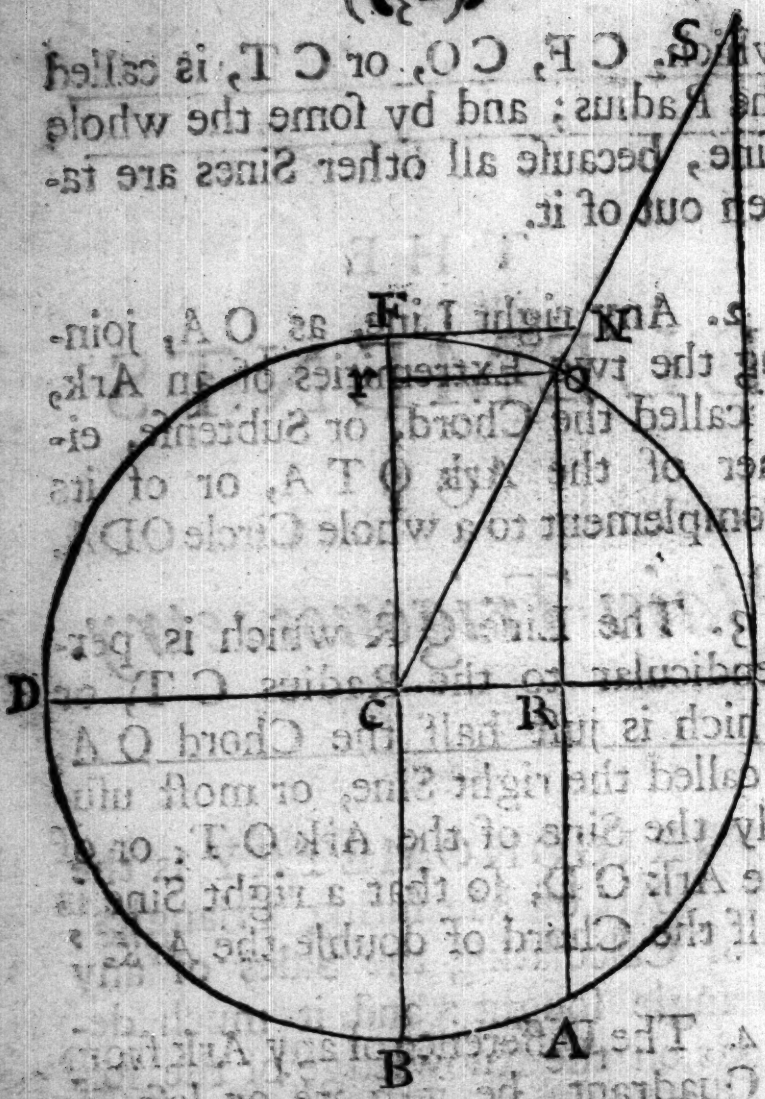
The Reaſon of its Publication was, that it might be in the Hands of my Auditory at the Publick Mathematick Lecture in Birchin-Lane, *when I am explaining this Subject to them; and where as ſoon as I have finiſhed* Plain Trigonometry, *I ſhall proceed to Read* Spherick

rick Geometry, *and* Projection of the Sphere in Plano, *and then* Spherick Trigonometry; *which last Science cannot be well understood, without the* Learner *go in the Order and Method I have now mentioned. And this Hint may serve to shew the great defect of such Treatises of the whole Art of* Trigonometry, *as have nothing of* Spherick Projection *Introductory to the Doctrine of* Spherick Triangles.

THE

THE ELEMENTS OF *Plain Trigonometry.*

TRIGONOMETRY is the Art of Meaſuring Triangles, or of Calculating the Sides of any Triangle ſought: and it much depends on the Knowledge of the following Lines in and about a Circle.



DEFINITIONS.

1. The Line D T in a Circle is called the Diameter; one half of which,

which, C F, C O, or C T, is called the Radius; and by ſome the whole Sine, becauſe all other Sines are taken out of it.

2. Any right Line, as O A, joining the two Extremities of an Ark, is called the Chord, or Subtenſe, either of the Ark O T A, or of its Complement to a whole Circle O D A.

3. The Line O R which is perpendicular to the Radius C T, or which is juſt half the Chord O A, is called the right Sine, or moſt uſually the Sine of the Ark O T, or of the Ark O D, ſo that a right Sine is half the Chord of double the Ark.

4. The Difference of any Ark from a Quadrant, be it more or leſs, is called its Complement. Thus O T is the Complement of the Ark O F, being what it wants of being a Quadrant;

drant; and 'tis alſo the Exceſs by which the Ark O D exceeds a Quadrant; the Line $r\rho$ is the right Sine of the Complement, and therefore 'tis called the Co-Sine or Sine-Complement of the Ark O T; as O R is the Co-Sine of the Ark F O.

5. A Line, as T S, touching the Circle in the Point T, and perpendicular to the Radius C T, is called a Tangent: and if a right Line be drawn through O from the Centre of the Circle C, meeting with the Tangent in S, that Line S C is called a Secant, and they both limit one another, ſo as to be the Tangent and Secant of the Ark O T: F N is the Co-Tangent, and C N the Co-Secant of the ſame Ark; but they are the Tangent and Secant of the Complimental Ark O F.

6. The

6. The Line R T intercepted between the Right Sine and the Tangent, is called the Verſed Sine, and by ſome *Sagitta*.

7. Whatever Number of degrees an Ark wants of a Semi-Circle, is called its Supplement.

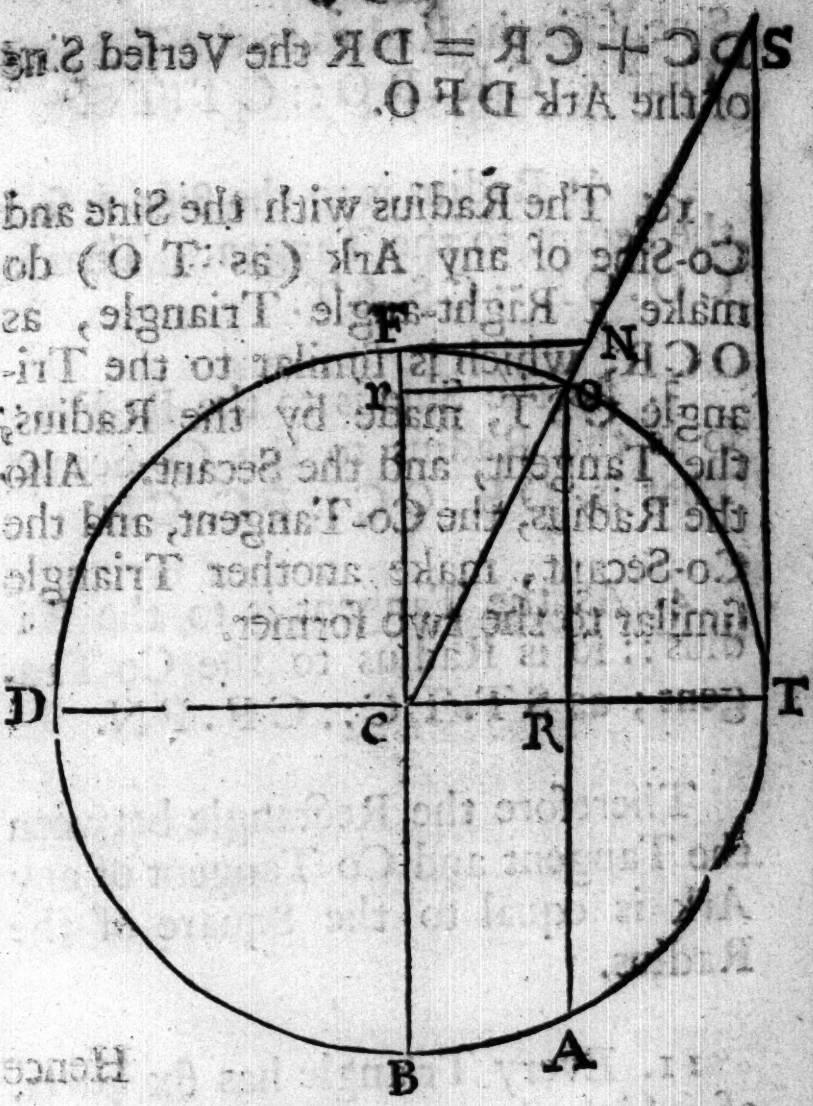
8. The Line C R, which is the part of the Radius C T lying between the Centre and the right Sine, is always equal to, and may be taken for the Co-Sine, O; and C r is equal to the right Sine O R.

9. If any Ark be leſs than a Quadrant, as T O, the Difference between the Radius and the Co-Sine is the Verſed Sine R T; but if it exceed a Quadrant, as the Ark D F O doth, then the Sum of the Radius and Co-Sine is the Verſed Sine: Thus

DC + CR = DR the Verſed Sine of the Ark DFQ.

10. The Radius with the Sine and Co-Sine of any Ark (as TO) do make a Right-angle Triangle, as OCR, which is ſimilar to the Triangle CST, made by the Radius, the Tangent, and the Secant. Alſo the Radius, the Co-Tangent, and the Co-Secant, make another Triangle ſimilar to the two former.

Hence



Hence 'tis plain,

1. That as the Co-Sine is to the

Sine, ſo is the Radius to the Tangent, That is, CR.RO::CT.TS.

2. As Radius is to the Sine :: ſo is the Secant to the Tangent. That is, CO.OR::CS.ST.

3. As the Sine is to the Radius :: ſo is the Radius to the Co-Secant. That is, OR.OC::FC.CN.

4. As the Tangent is to the Radius :: ſo is Radius to the Co-Tangent; as ST.TC::CF.FN.

Therefore the Rectangle between the Tangent and Co-Tangent of any Ark is equal to the Square of the Radius.

11. Every Triangle has ſix parts, of which three are Sides and three Angles; and of theſe if we have three given, we can find the reſt, (except in

in the Caſe where the three Angles only of a plain Triangle are given.)

For from thence the Sides cannot be found, becauſe two Triangles may be equiangular, and yet have the Sides by no means of the ſame length.) We can find the reſt, I ſay, if ſuppoſing the Radius divided into any Number of equal parts, we can but diſcover how many of ſuch parts any Sine, Tangent, or Secant of any Ark or Angle doth contain. Now this is ready done to our hands, in the Tables of Sines, Tangents, and Secants, which we have, with prodigious Induſtry, in Books ready calculated for this purpoſe.

12. When therefore any Triangle is given to be reſolved, the firſt thing we have to do is to conſider, that there is in the Table of Logarithms, Sines, Tangents, and Secants, a Tri-

Triangle exactly ſimilar and equal to that which we are required to ſolve, and whoſe Sides are to one another in the very ſame proportion of thoſe of the Triangles propoſed.

2. We muſt underſtand whatever *Ratio* one Side of the Triangle given hath to the other Side about the ſame Angle, conſidered as Lengths eſtimated or number'd by any known meaſure, as ſuppoſe Inches, Yards, Miles, Leagues, *&c.* the very ſame hath the two Sides about the ſame Angle in the Triangles in the Tables, or in the Tabular Parts: which two things well underſtood, do lead us into the whole Myſtery of Trigonometrical Calculations.

13. Trigonometry is either Plain or Spherical; and both may be reſolved by the means of four Proportions, which becauſe of their excellent Uſe are called Axioms. 14.

14. The firſt of which relates to Rectangle plain Triangles; and is this.

Axiom I.

In a Right-angle Triangle, if either of the Legs be ſuppoſed to be the Radius of a Circle, the other Leg will be the Tangent of the oppoſite Angle, or of the Angle at the Centre; and the Hypothenuſe will be the Secant of that Angle: But if you imagine the Hypothenuſe to be the Radius of a Circle, then each Leg will be the Sine of its oppoſite Angle, or of the Angles at the Centre; as is plain from the adjoining Figures.

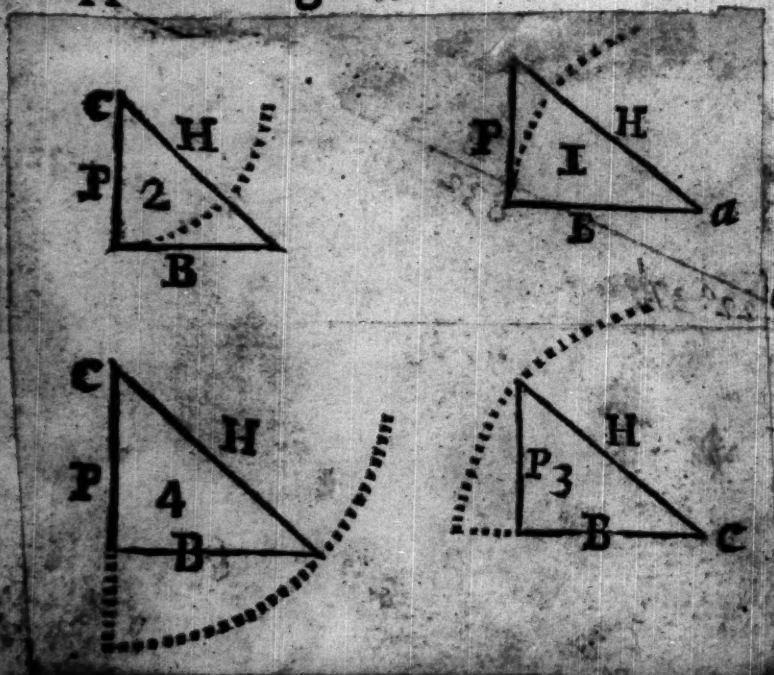
In the firſt of which, B (the Baſe) being made the Radius, P (the Perpendicular) is the Tangent of the Angle at the Centre of the Circle; *a* which is oppoſite to P and the Hypothenuſe,

pothenuſe, is the Secant of the ſame Angle.

In the ſecond Figure, where P is made the Radius, B is the Tangent of the oppoſite Angle at the Centre *c*.

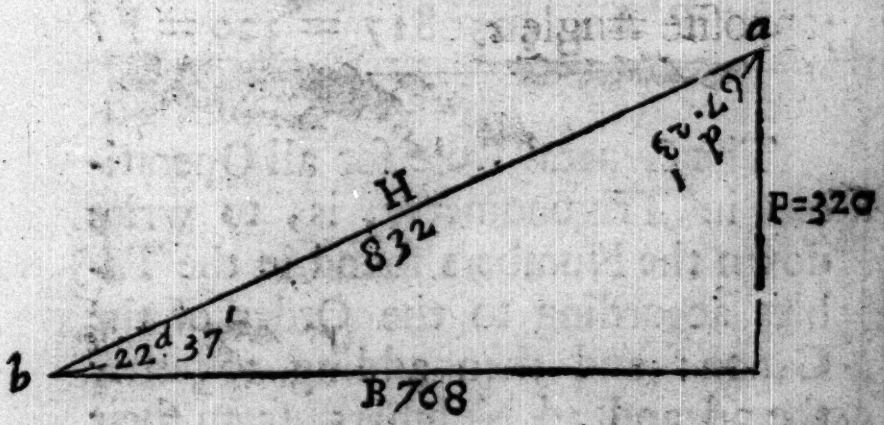
In the third Figure, where H the Hypothenuſe is made the Radius, P is the right Sine of the oppoſite Angle at the Centre. And,

In the fourth Figure, H being alſo made Radius, but C the Centre of the Circle, B will be the Sine or its oppoſite Angle *c*.



The Seven Cases of Plain Triangles.

	Given	required	Proportions	given	required.
1	B, *a*, *b*	P	R. B :: Tan. *b*. P	*c*. √ √	*c*
2	B, *a*, *b*	H	S, *a*, B :: R. H	*c*. √ √	*b*
3	B. H	√ √	H. R :: B. S, *a*	*c* √. *b*	√ √
4	B, H	P	H. R :: B. S, *a* Then R. T. *b* :: B. P Or R. B :: T. *b*. P	which finds the √ *a*,	
5	B. P	√ √	B. R :: P. T. *b*	*c* √.	√ √
6	B. P	H	B. R :: P. T. *b* Then S. *b*. P :: R. H	by the laft. *c* √.	*b*
7	H √ √	B	R. H :: S, *a*. B	*b* √ √	*c* √



The

The Seven Cases of Right-angle Plain Triangles.

CASE I.

Given B. *a.* and *b.* Required P?

Canon | R. B :: T. *b.* P? |

R = 10.
B = 2.8853612 = 768
T, *b* = 9.6197205 = 22°. 37′.
P = 2.5050817 = 320 = P?

The General Rule for all Operations in Trigonometry, is, to write down the Numbers found in the Tables according to the Order of the Canon: and then adding together the 2d and 3d Numbers, from their Sum subſtract the 1ſt, the Remainder

der is the Logarithm of the Term fought.

By Gunter's *Line*: Extend the Compaſſes from 45 deg. on the Tangents to 22 deg. 37 min. the ſame Extent will reach in the Line of Numbers from 768 backward to 320.

CASE II.

Given B, *a*, *b*. Req. H?

S, *a*. B :: R. H.

S, *a* = 9. 9652480 = 67°. 23'.

B = 2. 8853612 = 768.

R = 10.

H = 2. 9201132 = 832 = H.

By Gunter's *Scale*: Extend the Compaſſes from 67 deg. 23 min. to

90

90 deg on the Line of Sines; the ſame Extent will reach from 768 to 832 on the Line of Numbers.

The general Method to ſtate any Caſe, or to form the Canon.

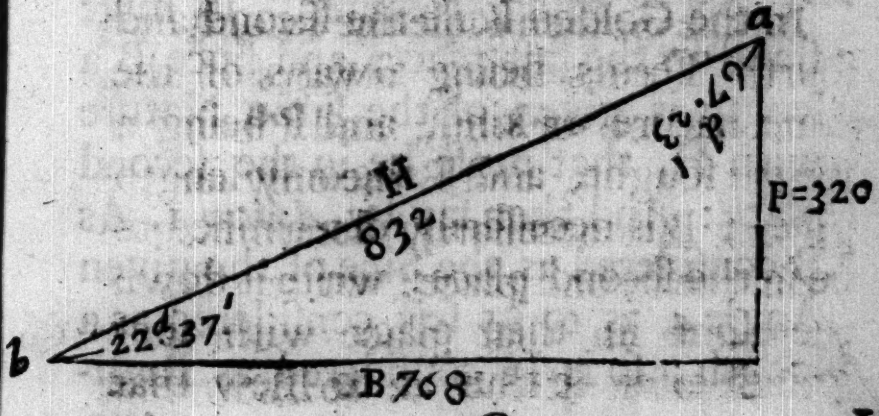
Conſider that the thing ſought muſt always ſtand in the 4th or laſt Place: and therefore *in Caſ.* 1. ſince P, a length is ſought, that muſt be the laſt of the Four Terms; place it therefore laſt with an Interrogation-point after it, to ſhew that it is required or ſought.

In the Golden Rule the ſecond and fourth Terms being always of the ſame nature or kind, and P being a length ſought, and B the only length given; B is neceſſarily determined to be in the ſecond place: write it down therefore in that place with four Points after it thus :: to ſhew that the

the Proportion disjoins or breaks off there.

3. Consider that the Hypothenuse not being either given or sought, the first Axiom determines you to work by Tangents: and the Side given B being supposed Radius, the Proportion must be; As B consider'd, as Radius is to its self considered as a Length given :: So will P considered as the Tangent of the Angle *b*, be to its self considered as a Length sought. That is,

R. B :: T, *b*. P?



C If

If the Angle *b* had been ſought, that muſt have been placed laſt, and it would have ſtood thus; as in *Caſe* the 5th.

B.	R	::	P.	T, *b*?

But if the Hypothenuſe had been in the Queſtion, either given or required, you muſt have worked by Sines, and the Hypothenuſe will be always Radius; as in *Caſe* 2. where H is ſought.

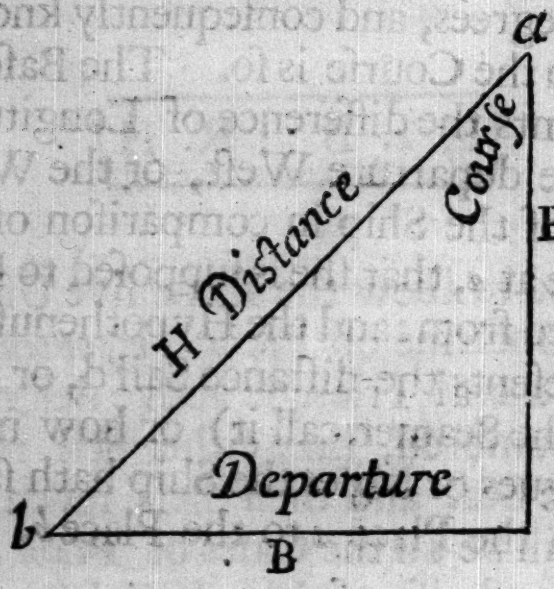
For H being required, it muſt ſtand in the laſt place, and ſince B a length is given of the ſame Nature with H, that muſt be in the ſecond place: And then ſay by *Axiom* 1. As B conſidered as the Sine of the given Angle *a*, is to its ſelf conſidered as a length; ſo is H conſidered as Radi-

us,

us, to its ſelf as a length ſought. That is,

| S, *a*. B :: R. H |

And to ſhew the Extenſive Uſe of this Doctrine of *Plain Trigonometry*, the General Triangle above deſcribed and numbred, may either firſt relate to the Sea, and then theſe ſeven Caſes will be all the Caſes of *Plain Sailing*; and alſo of Mr. *Wright*'s, or as 'tis commonly tho' falſly called, *Mercator*'s *Too*; regard being firſt had to the way of working by Meridional parts, *&c.*

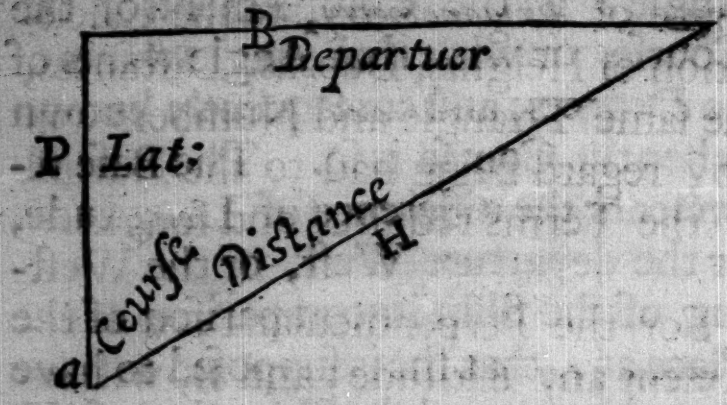


For, in this Triangle the Angle *a*, is the Angle of the Rhumb, or the Angle which the Line of the Ships Courſe makes with the Meridian, and therefore uſually by the Sailers called, *The Courſe*; and the Side P repreſents the Meridian of any Place, acroſs which the Ship is ſuppoſed to ſail; and conſequently on it muſt be accounted the difference of Latitude between

between the two Places *a* and *b*. The Angle *b* is the Complement of the Courſe, or what that Angle wants of 90 Degrees, and conſequently known when the Courſe is ſo. The Baſe repreſents the difference of Longitude, or the departure Weſt, or the Weſting of the Ship in compariſon of the Place at *a*, that ſhe is ſuppoſed to have parted from: and the Hypothenuſe H, repreſents the diſtance Sail'd, or *Run* (as the Seamen call it) or how many Leagues or Miles the Ship hath ſailed from the Place *a* to the Place *b*.

This being underſtood, if this firſt Caſe be made a Caſe of *Plain Sailing*, there will be given Courſe and Departure; required difference of Latitude, and the Canon is.

As Radius is to the Departure in Miles :: So is the Co-tangent of the Courſe, to the Difference of Latitude in Miles.



N. B. Here the Ship being at *a*, is ſuppoſed to Sail to the South Weſtward (or to ſpeak exactly, W.S.W.) and therefore her difference of Latitude is reckoned to the South, and her departure to the Weſt: But by Inverting the ſame Triangle, you may ſuppoſe the difference of Latitude North, and the Departure Eaſt; for the North is accounted to lie right before you, and conſequently the Eaſt to the Right-hand, and the Weſt to the Left; wherefore the Courſe now is E. N. E.

2. If

2. If you would apply the Doctrine of *Trigonometry*, to the Calculation of Heights, Depths, Distances, the same Triangle and Numbers will do; regard being had to the Nature of the Terms required and sought.

For the Perpendicular P, will represent any Altitude; and B, will represent a Distance from the Foot of it measured on the Ground: The Angle at *b* is found by the Quadrant or some such Instrument; and consequently you may find P by *Case* 1. For

As Radius is to the Distance, from the Foot of the Object :: So is the Tangent of the Angle of Altitude, to the Height of the Place, *i. e.*

R. B :: T, *b*. P.

Or, ſuppoſe the Diſtance B were required from above, by taking the Angle *a*, and finding the Length of P, by a String and Plummet. Then will

R. P : : T, *a*. B

This being promiſed as to the general Uſe of *Trigonometry*, let us proceed to

CASE III.

Given B, H. Req. the Angles *a*, *b*.

H. R : : B. S, *a*

H = 2.9201233 = 832.

R = 10.0000000

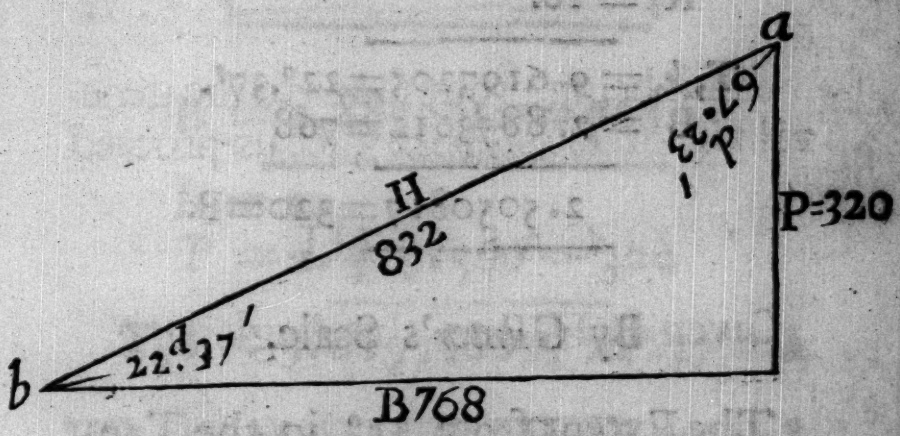
B = 2.8853612 = 768.

S, *a* = 9.9652379 = 67°. 23'.
= Angle *a*.

Which

Which Subtracted from 90°, gives the Angle $b = 22°. 37'$.

By *Gunter*'s Scale, the Extent from 832, back to 768, in the Line of Numbers, will reach from 90°. to 67°. 30'. in the Line of the Sines.



CASE IV.

Given B, H, Req. P.

Having found *a*, by the foregoing Case, this will be the Theorem.

| R. B :: T. *b*. P. |

Or,

Or,

| R. T, *b* :: B P |

R = 10.

T, *b*, = 9. 6197205 = 22°. 37′.
B = 2. 8853612 = 768

2. 5050817 = 320 = P.

By *Gunter*'s Scale.

The Extent from 45°. in the Tangent-line, back to 22°. 37′. will reach in the Line of Numbers from 768, back to 320.

CASE

CASE V.

Given B, P. Req. ✓ ✓.

B. R :: P. T, *b*

B = 2.8853612 = 768

R = 10.

P = 2.5051500 = 320

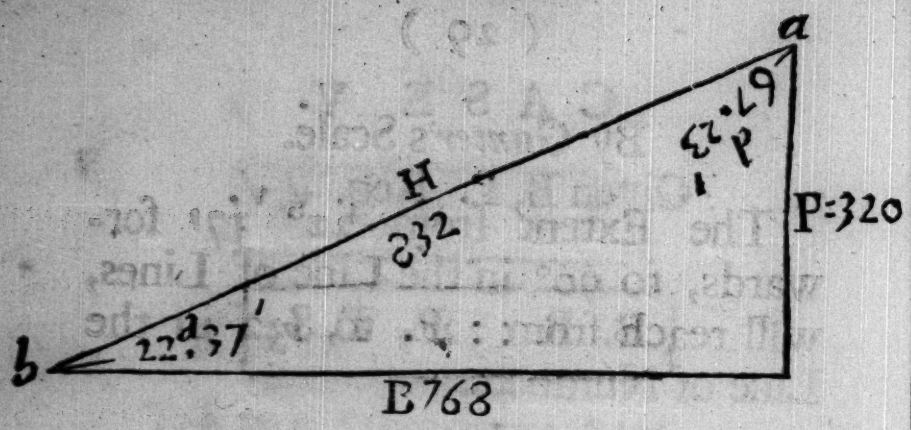
T, *b* = 9.6197888 = 22°.37′ = *b*.

Which 22°. 37′ Subtracted from 90°. leaves *a* = 67°. 23′.

By *Gunter*'s Scale.

The Extent from 768 back to 320, in the Line, will reach from 45 back to 22°. 37′. in the Tangents.

CASE



CASE VI.

Given B, P, Required H.

Having found *b* by the foregoing Case, this will be the Theorem.

| S, *b*. P :: R. H. |

S, *b* = 9. 5849685 = 22° 37'

P = 2, 5051500 = 320

R = 10,

H = 2, 9201815 = 832 = H?

By

By *Gunter*'s Scale.

The Extent from 22° 37′ forwards, to 90° in the Line of Lines, will reach from 320, to 832 in the Line of Numbers.

CASE VII.

Given H and ✓✓, Required B?

| R. H :: S, *a*. B |

R, = 10,

2, 9201233 = 832

S, *a* = 9, 9652480 = 67° 23′

B = 2, 8853719 = 768 = B.

By

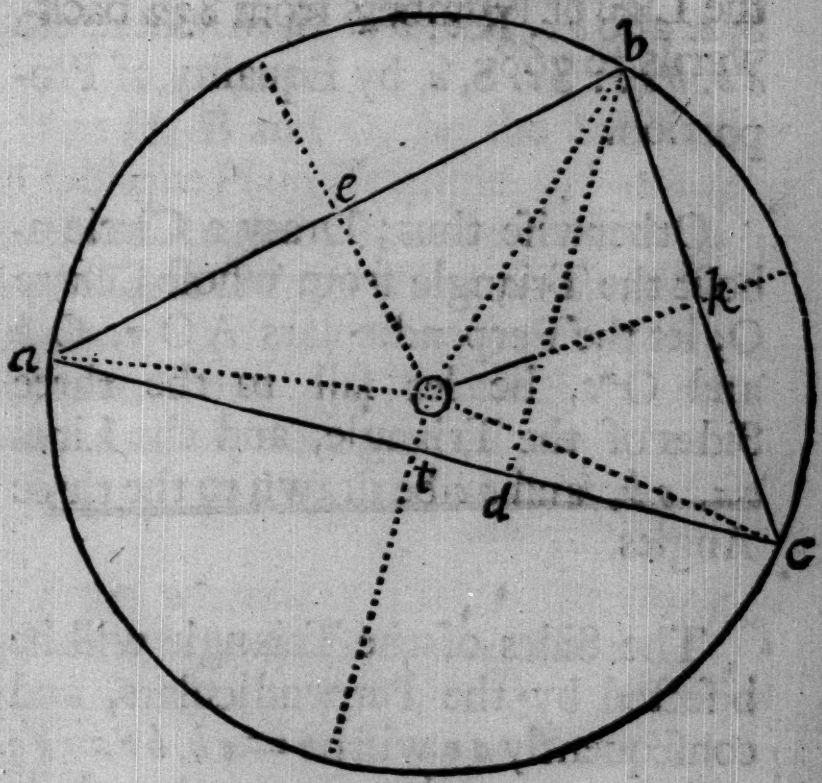
By *Gunter*'s Scale.

The Extent from 90° in Line of Sines back to 67° 23′ will reach in the Line of Numbers from 832 backwards to 768.

AXIOM

The Resolution of Oblique Plain Triangles.

Axiom II:



In every Triangle a, b, c, *the Sides are in proportion to one another, as the Sines of their opposite Angles.*

Let

Let fall from the Angle *b*, a Perpendicular to the Baſe *a c*: For then the whole will be reſolved into two Right Angle Triangles; and conſequently by Axiom the firſt *a b*. R: : *b d*. S, *a*. alſo *b c*. R : : *b d*. S, *c*. wherefore *a b*. *b c* : : S *c*. S, *a*. by Equality of Proportion.

Otherwiſe thus; Draw a Circle about the Triangle from whoſe Centre O, let the Perpendiculars A O *e*, O *k* and O *t*, be let fall to the three Sides of the Triangle, and the Lines *o a*, *o b*, and *o c* be drawn to the three Angles.

The Sides of the Triangle will be biſected by the Perpendiculars, and conſequently *a e* will be = *e b*, *b k* = *k c* and *c t* = *t a*. wherefore as the whole Line *a b* is to *b c* : : ſo will the half ſide *a e* be to the half *k c*; but *a e* and *k c*, are the Sines of the Angles at the Centre

Centre *a o e* and *k o c*, which Angles at the Centre are ſeverally equal to the Angles of the Triangle *c* and *a*, becauſe they ſtand on half the Arks, that the Angles of the Triangle do; wherefore *a b. b c* :: S, *c*. S, *a*.

Let H, B and O, be the Sides of an Oblique Plain Triangle *a*, *b* and *c*, its three Angles; here becauſe there is no Right Angle, three things muſt be always given; that is, either two Sides and one Angle, one Side and two Angles, or elſe all the three Sides.

From whence will arrive the three former of the ſix following Caſes.

	Given.	Required.	Proportions.
1	H. O. *c.*	*b.*	O, S *c* :: H. S, *b.* Note that the Angle *b* is ambiguous, and you muſt collect from the Circumſtances of the Triangle, whether it be Obtuſe or Accute.
2	H. O. *c.*	B	Here firſt find the Angle *b* by *Caſe* 1, thence *a* will be known by taking the Sum of *b* + *c.* from 180°. Then S, *c.* H :: S, *a.* Or S, *b.* O :: S, *a.* B.
3	*c*, *b*, O,	H.	S, *c.* O :: S, *b.* H.
4	H. *a.* O.	*c*, *b.*	H + O. H — O :: T $\frac{1}{2}$ Z of the oppoſite Angles to T $\frac{1}{2}$ X of the oppoſite Angles, and $\frac{1}{2}$ Z + $\frac{1}{2}$ X = *b.* and $\frac{1}{2}$ Z — $\frac{1}{2}$ X = *c.*
5	H. *a*, O.	B.	Find the Angles by the former Caſe, and then S, *b.* H :: S, *a.* B. or S, *c.* O :: S, *a.* B.
6	H. O. B. 3 Sides.	3 Angles.	B. H + O :: H — O. X. and then $\frac{1}{2}$ B + $\frac{1}{2}$ X = C B and $\frac{1}{2}$ B — $\frac{1}{2}$ X = B *b.* Then will H. R :: C B. Co-ſine Angle C. and O. R :: *b* B, Co-ſine Angle *b.* Axiom 1.

And the three laſt are ſolved by the help of the two following Axioms.

Axiom III.

As the Sum of the Legs about the Angle given, is to their Difference : : So is the Tangent of half the Sum of the other two Angles, to the Tangent of half their Difference..

Now the Sum of the other two Angles is known, being what the given Angle wants of 180 Degrees and their Difference is now found; add therefore their half Sum, and half Difference together, and it gives you the *greater of the two Angles ſought*; and half the Difference Subtracted from the half Sum leaves the leſſer Angle ſought. And thus having found the Angles; if the ſide oppoſite to the former given Angle be ſought, it will be found eaſily, by *Pardie's Axiom*, That the Sides are as the Sines of the Angles.

The Demonſtration of the Third Axiom is briefly thus,

DEMONSTRATION.

I ſay the Sum of the Legs of any Angle *a*, is to their Difference : : as the Tangent of ½ the Sum of their oppoſite Angles, is to the Tangent of half their Difference.

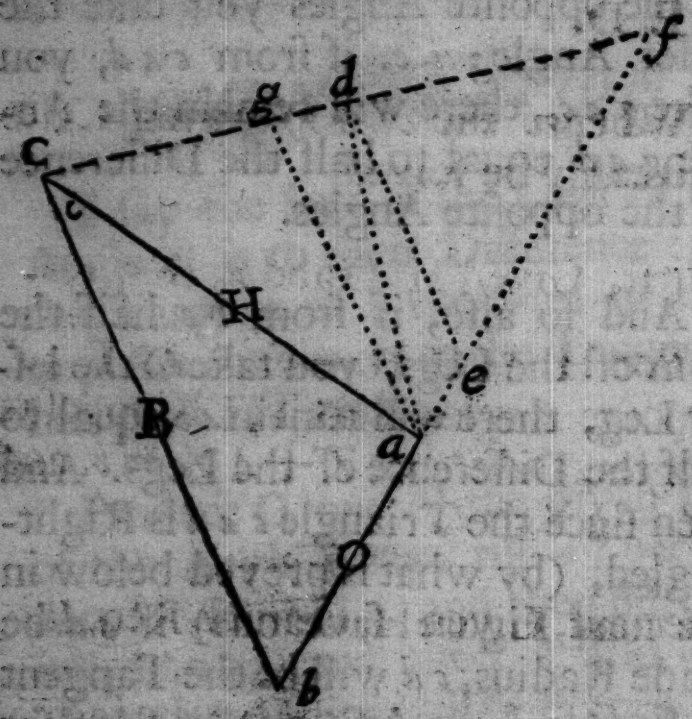
Produce O one of the given Legs of the Angle given, till *a f* become equal to H or C *a*, and then biſect *b f* in *e*, join *c f*, and biſect it alſo in *d*: Draw *a d*, which will be Perpendicular to *c f* (2. 16.) and draw *d e*, which will be Parallel to *c b*: (6. 92.) Then will the Angle *c a d* = *d a f*: i. e. to the half of *c a f*. which External Angle *c a f* = *c* + *b*: That is, to the Sum of the oppoſite Angles required.

Draw

Draw then *g a*, Parallel to *c b*; ſo will the Angle *g a c* be equal to the alternate one *c*. And if from $\frac{1}{2}$ the Sum of the oppoſite Angles you take the leſſer Angle: *i. e.* if from *c a d*, you take *g a c*, there will remain the Angle *g a d* equal to half the Difference of the oppoſite Angles.

And ſo alſo, if from *b e* half the Sum of the Legs, you take O the leſſer Leg, there will remain *a e* equal to half the Difference of the Legs. And then ſince the Triangle *c a d* is Right-angled, (by what is proved below in the next Figure ſave one) if *a d* be made Radius, *c d* will be the Tangent of the Angle *c a d* (i. e. the Tangent of $\frac{1}{2}$ Sum of oppoſite Angles;) and in the little Triangle *g a d*, *g d* will be Tangent of the Angle *g a d* (i. e. the Tangent of $\frac{1}{2}$ Difference of oppoſite Angles.) But the Segments of the

Legs of any Triangle cut by Lines Parallel to the Baſe being Proportional, *eb. ea :: cd. dg*; That is in



words, *Half the Sum of the Legs is to half their Difference :: as the Tangent of* $\frac{1}{2}$ *the Sum of the oppoſite Angle, is to the Tangent of* $\frac{1}{2}$ *their Difference*; but Wholes are as their Halves: Where-

fore

fore the Sum of the Legs is to their Difference :: As the Tangent of $\frac{1}{2}$ the Sum of the oppoſite Angles, is to the Tangent of half their Difference. Q. E. D.

Whence the two following Caſes will eaſily be ſolved.

Firſt, Given H, O and *a*, Req *c*, *b*.

For H + O H − O :: As T, $\frac{1}{2}$ Z oppoſite Angles, is to T, $\frac{1}{2}$ X oppoſite Angles, and then $\frac{1}{2}$ Z + $\frac{1}{2}$ X = *b*. and $\frac{1}{2}$ Z − $\frac{1}{2}$ X = *c*.

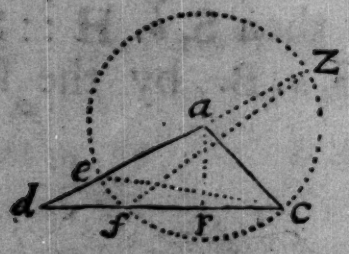
Second, Given H, O and *a*, Req. B?

Firſt find the Angles by the former Caſe, and then S, *b*. H :: S, *a*. B : or S, *c*. O :: S, *a*. B. by the ſecond Axiom.

Axiom. IV.

The Baſe is to the Sum of the Legs :: as the Difference of the Legs is to the Difference of the Segments of the Baſe made by a Perpendicular let fall from the Angle oppoſite to the Baſe.

For, there is alſo another Caſe in plain Oblique Triangles, which requires a particular Axiom to ſolve it; and that is, where All three Sides are given to find the Angles. Here let fall a Perpendicular from any Angle to its oppoſite Side as *a p*; and then



ſay,

ſay, As the Side $d\,c$ is to $d\,a + a\,c$ the Sum of the other two Sides :: So is the Difference of thoſe two Sides $d\,a - a\,c$ to a fourth Number. Half of which added to $\frac{1}{2}\,d\,c$, gives you the Segment of the Baſe $d\,p$; and if Subtracted from $\frac{1}{2}\,d\,c$, it will leave the other Segment $p\,c$. And when thoſe Segments are thus found, the Angles are eaſily had thus: $d\,a$. *Radius* :: $d\,p$. Co-ſine of the Angle d. And $a\,c$. *Radius* :: $p\,c$. Co-ſine of the Angle C.

The Demonſtration of which laſt Axiom is this,

DEMON-

DEMONSTRATION.

On the Centre *a*, with the Diſtance *a c* deſcribe a Circle, which will Interſect both the other Sides of the Triangle, and then *z d* will repreſent the Sum of the Legs *d a* and *a c*: *a e* will repreſent their Difference, and *d f* will repreſent the Difference of the Segments of the Baſe made by the fall of the Perpendicular *a* P.

Then I ſay, *d c. d z* :: *d e. d f*. That is, *The Baſe is to the Sum of the Legs* :: *as the Difference between the two Sides is to the Difference of the Segments of the Baſe*; as is apparent from *Prop.* 67. of *Pardie*'s Sixth Book, after drawing the Prickt Lines *e c* and *f z*.

And then the Caſe will ſtand thus,

3, Given

3, Given H, O, B, all the Three Sides, Required the Angles. See the Figures before.

I say by this Axiom B. $H + O :: H - O. X$, which expresses the Difference of the Segments of the Base, $= df$ in the Figure.

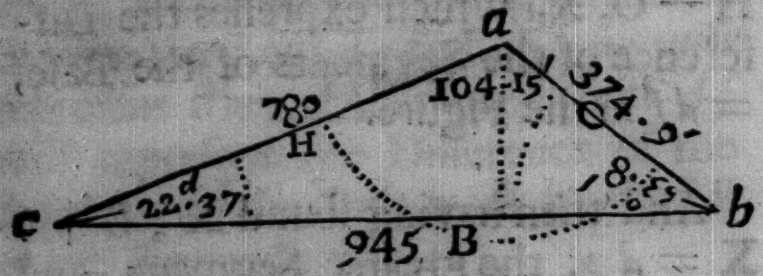
And then having found $X \frac{1}{2} B + \frac{1}{2} X = d\,p$, the greater Segment, and $\frac{1}{2} B - \frac{1}{2} X = p\,c$ the lesser, by which means the two Right-angled Triangles $a\,d\,p$ and $a\,p\,c$ will be solved easily: For $d\,a$. *Radius* :: $d\,p$. Co-sine of the Angle d. and $a\,c$. *Radius* :: as $p\,c$. to the Co-sine of the Angle c.

The

The Operation of the Six Cases of Oblique angled Triangles.

CASE I.

Given H, O, *c*, Required *b*.



O. S, *c* : : H. S, *b*.

The Numbers of the Triangle are as follows.

$\sphericalangle c = 22°.37'$
$\sphericalangle b = 53.08$
$\sphericalangle a = 104.15$ Or, its Suppl. $= 75°.45'$

H = 780
O = 374. 9
B = 945

O

O = 2.5739154 = 347.9

S, c = 9.5849685 = 22°. 37
H = 2.8920946 = 780

Sum = 12.4770631

S, b = 9.9031477 = 53°. 8'

CASE II.

Given H, O, c. Required B.

S, c. O :: S, a. B.

S, c = 9.5849685 = 22°. 37'

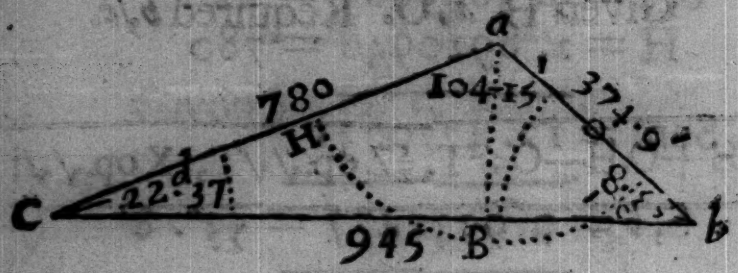
O = 2.5739154 = 374. 9
S, a = 9.9864273 = 104°. 15'

Sum = 12.5603427

B = 2.9753742 = 945

CASE

CASE III.



Given *c*, *b*, O. Required H.

S, *c* = 9.5849685 = 22°. 37'

O = 2.5739154 = 374. 9

S, *b* = 9.9031084 = 53°. 8'

Sum = 12.4770238

H = 2.8920553 = 780

CASE

CASE IV.

Given H, *a*, O. Required *b*, *c*.

H+O. H−O :: T, ½Z op. √√ T, ½X op. √√

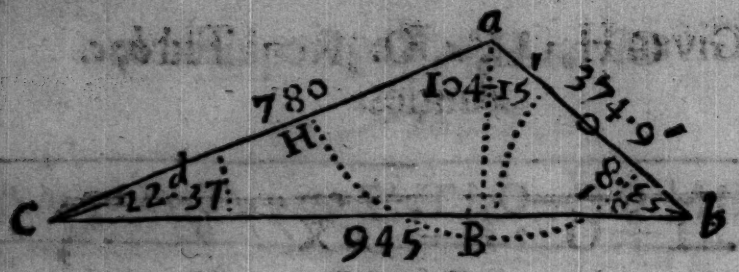
H+O = 1154. 9
H−O = 405. 1
½ Z opp. ∠∠ = 37°. 52′

H+O = 3.0625820 = 1154 9
H−O = 2.6074550 = 405. 1
T ½ Z = 9.8907254 = 37°. 52′
Sum = 12.4981804
T, ½ X. 9.4355984 = 15°. 15′

Then will ½Z + ½X = 53° 7′ = *b*:
and ½Z − ½X = 22° 37 = *c*.

CASE

CASE V.



Given H, *a*, O. Required B.

Find the Angles *b* and *c*, by *Case* 4. then

S, *b*. H :: S, *a*. B

S, *b* = 9.9031084 = 53°. 8

H = 2.8920946 = 780

S, *a* = 9.9864273 = 104°. 15′ *

Sum = 12.8785219

B = 2.9754135 = 945

* This is the Sine of 75. 45, the Supplement of 104.15, to a Semicircle.

CASE

CASE VI.

Given H, O, B. Required Three Angles.

B. H + O :: H − O. X.
½ B + ½ X = CB & ½ B − ½ X = Bb
H. R :: Bc. ∽ c : & O. R :: Bb. ∽ b

B = 2.9754318 = 945.

H + O = 3.0622058 = 1154.9
H − O = 2.6074540 = 405.1

½ B = 472 ½ 5.6696958 =
½ X = 247 ½

2.6942280 = 495

$\frac{1}{2}B+\frac{1}{2}X=720$
$\frac{1}{2}B-\frac{1}{2}X=225$

H = 2.8920553 = 780′
R = 10.
BC = 2.8573225 = 720
Σc = 9.9652672 = 22°37′

O = 2.5739154
R = 10.
B*b* = 2.3521825
Σ*b* = 9.7782671 = 53°08′

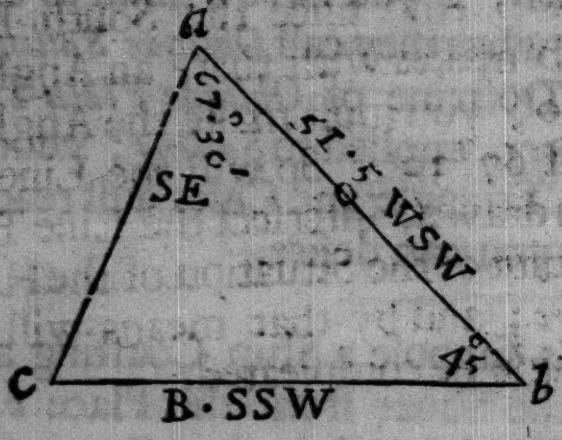
And when thus the Nature, Reaſon, and Method of Operation in the Calculating of the Sides and Angles of Oblique Triangles, is fully underſtood; its Application, Uſe, and Practice, will be very eaſie and plain, tho' its extent be very large and ample. For,

Firſt,

First, If you have a mind to apply it to what they call *Oblique Sailing*, or the Doctrine of *Oblique Plain Trigonometry*, applied to *Sailing*; your former Triangle and Cases will reach all that you can desire.

As suppose a Ship Coasting along by the Shoar from the Place *b*, sets an Head Land by her Compass (as *c*) and finds it to bear from her S.S.W. then she sails on, W.S.W. 51.5 Miles or Minutes to *a*; and then finds that the Head Land bears from her full S.E.

'Tis required to determine her distance from this Head Land when she was at *b*, and now she is at *a*.



First, To Plot the Triangle.

Draw a ſtrait Line as B, repreſenting the firſt bearing of the Head Land, which was S. S. W. Then becauſe the Ship Sail'd Weſt South Weſt, make the Angle *b*, equal to 45°, and ſo will the Line O repreſent the Diſtance ſailed, and *b* will be the Courſe. Prick off the Diſtance run, *viz.* 51. 5 Miles from *b* to *a*. Then becauſe the Ship ſailed W. S. W. the contrary Rumb from *a* to *b*, muſt be E. N. E.

ENE: And ſince the South Eaſt Rumb makes with that, an Angle of 67° 30′, you muſt make the Angle at *a*, juſt 67° 30′. So will the Line H, when drawn, interſect the Line B in the Point of the Situation of the Head Land *c*; and by that means will the Triangle be compleated.

This being done, if you pleaſe, (tho' plotting the Triangle, is not of abſolute neceſſity, but very uſeful and inſtructive.) Then conſider, that it muſt be a Caſe of *Oblique Plain Triangles*, where is given the Angles *a* and *b*, and the Side O. Required H and B.

To find H ſay, (becauſe *c* is alſo known if *a* and *b* are.)

S, *c*. O :: S, *b*. H

Operation.

S, *c* = 9.9656153 = 67° 30′

O = 1.7118072 = 51.5 Miles,

S, *b* = 9.8494850 = 45°

11.5612922

H = 1.5956769 = 39.4 Miles.

And to find B, you may ſay,

As S, *b*. H :: S, *a*. to B:

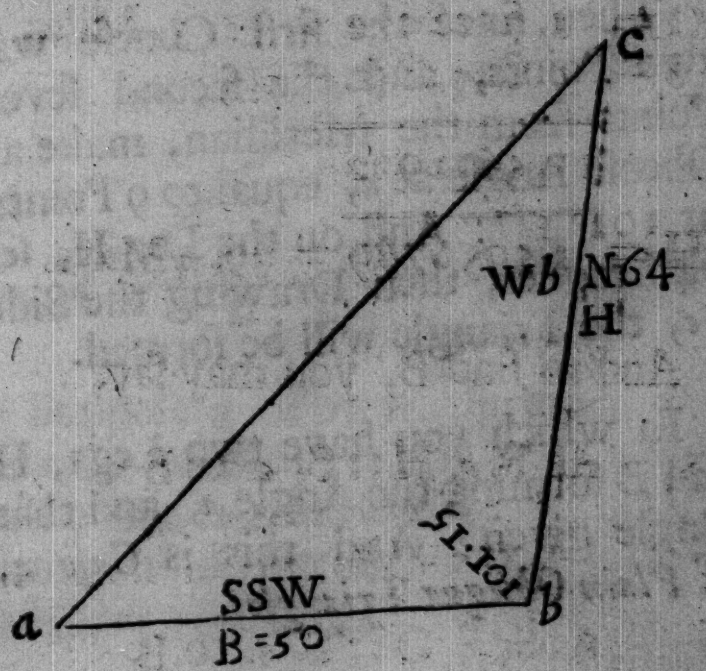
Or,

As S, *c*. O :: S, *a*. to B.

I ſhall

I ſhall give but one Inſtance more in this matter, which is this.

Suppoſe a Ship ſail S. S W. 50 Leagues, and then W. by N. 64 Leagues: What was her Direct Courſe, and what is her Diſtance from the Place ſhe went from?



To Plot the Caſe.

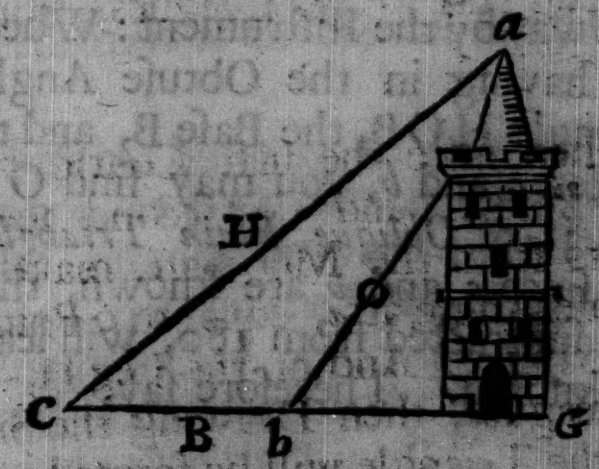
Let *b* be the Place from whence the Ship ſailed, and let B repreſent the S. S. W. Rumb, and the Diſtance thereon run = 50 Leagues, from *b* to *a*.

Then ſince the firſt Courſe was two Points, and the ſecond ſeven Points from the Meridian, make an Obtuſe Angle at *a*, equal to 9 Points, or 101° 15′: And on the Leg H, ſet 64 Leagues, then Drawing the Side *a c*, the Triangle will be formed.

In which you have two Legs, H and B forming the Angle *a*, and that Angle given. And this is *Caſe* 4. of *Plain Oblique Triangles*.

Secondly,

Secondly, If you would apply this Part of *Trigonometry* to the measuring of Inaccessible Distances, Heights, *&c.* at Land; the Practice will be very easie.



Suppose a Tower, Steeple, *&c.* as *a* G, whose Height you would take, but can measure no nearer then from *c* to *b*, but know the Length B *b*, is = 100 Yards.

Here

Here you can take the Angle *a b* G, with your Quadrant, and consequently the Obtuse Contiguous one, *c b a*, is known.

The Angle *c*, is known after the same way by the Instrument: Wherefore having in the Obtuse Angled Triangle H O B, the Base B, and the Angles *c* and *b*, you may find O by *Case* 3. of *Oblique Plain Triangles*: For since *c* and *b* are known, their Sum subtracted from 180, will leave *c a b* known. Therefore say,

As S, *a*. B : : S, *c*. O.

And when is thus known or found, having in the Right-angled Triangle, *b a* G, the Hypothenuse O, and the Angle *b*.

By

By *Case* the 7th of Right-angled Triangles.

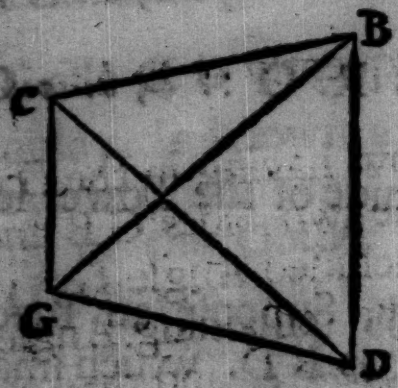
As R. O :: S, *b.* *a* G.

The Altitude of the Tower ſought.

Thirdly, Suppoſe an Inacceſſible Diſtance, as B D. which imagine to be the Diſtance between two Forts, Baſtions, *&c.* on the Wall, or Line of an Enemies City, or Camp; and that becauſe of the Cannon, *&c.* you can go no nearer then the Line G C, but can meaſure from G to C, and at each Station take Angles with an Inſtrument.

Having taken then by the Theodelite, *&c.* the Angles G C D, and C G D, and meaſured G C: And alſo having meaſured the Angles B G D, and B C G,

B C G, &c. This premiſed, which is eaſily done by the Inſtrument.



You have in the Triangle C B G, the Side C G, and the two Angles G, and G C B: Wherefore, alſo the Angle C B G. Say, therefore,

As S, B. is to G C :: S, C. to G B,

And conſequently G B is found.

Again, in the Triangle C D G, the Angles G C D, C G D, and the Side C G

C G being given, the Side G D will be found by this Proportion.

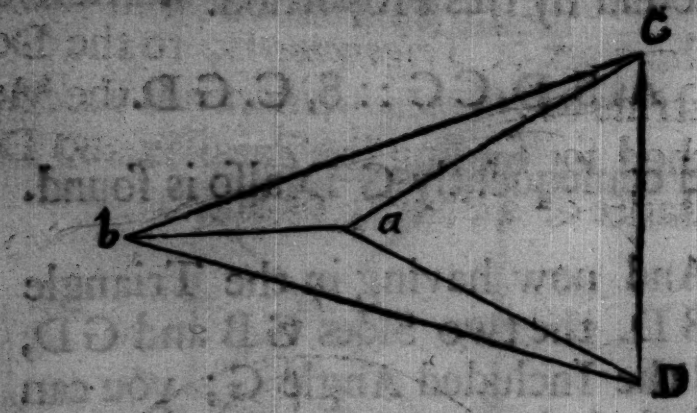
As S, D. C G :: S, C. G D.

And conſequently G D alſo is found.

And now having in the Triangle G B D, the two Sides G B and G D, and the Included Angle G; you can firſt find the remaining Angles ſeverally by *Axiom* 3. and then the Side B D by *Axiom* 2.

Or, If not being able to get nearer than the Point *a*, you could not meaſure ſideways as before, but only backward to *b*, or forward from *b* to *a*: You may then eaſily gain the length from D to B.

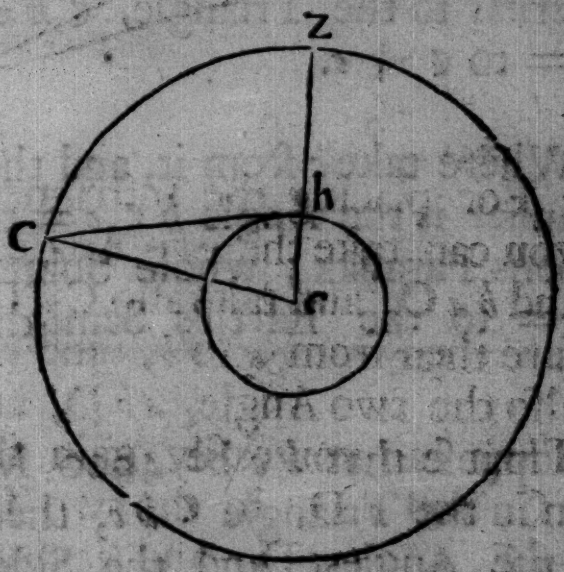
For



For placing the Inſtrument at *a*, you can take the two Angles *b a* D; and *b a* C, and take alſo C *a* D; meaſure then from *a* to *b*, and at *b* take alſo the two Angles *a b* D, and *a b* C. Then can you eaſily gain the Sides *a* C, and *a* D, in the two Triangles *b a* C, and *b a* D; and having before taken by the Inſtrument the Angle C *a* D, you may find the Side C D in that laſt Triangle *a* C D, by the ſecond and third *Axioms* of *Plain Oblique Trigonometry*. Alſo

Fourthly,

Fourthly, On the Application of this Part of *Trigonometry*, to the Doctrine of *Astronomy*, depends the Method for finding the Parallax and Distance of a Planet, or Star.



Let *c* be the Centre of our Earth, and *h* a Point on its Surface; at which, an Observer at *h*, takes with an Instrument, the Angle ☾ *h z*, or the

the Distance of the Meridian Moon ☾, from her Zenith at *z*.

Her True Distance from the Zenith, is known by the Astronomical Tables; which is the Angle ☾ *t z*: But the observed Angle ☾ *h z*, being external to the Triangle, ☾ *h z* will be = to ☾ + *c*.

Where take *c* from it, and the Remainder is the Angle *h* ☾ *c* = to the Moons Parallax; whose Subtense is *h c* = to the Earth's Semi-diameter.

Suppose that *h c* be 4000 Miles; then in the Triangle ☾ *h c*, there are all the Angles, and the Side *h c* known.

Wherefore,

As S, ☾. *h t* :: S, *h*. ☾. *t*.

Which

Which is the Moon's Diſtance from the Centre of our Globe.

And alſo, as

S. ☾. *h c* :: S, *c*. ☾ *h*.

The Moon's Diſtance from the Place of Obſervation.

F DIRE-

DIRECTIONS

For the USE of the

TABLES

OF

LOGARITHMS, SINES, TANGENTS, &c.

THESE Tables are in every ones Hands, and may be Bound up with this Treatise.

The

The Table of Logarithms ſerves to afford you Artificial Numbers to be uſed inſtead of thoſe Natural Numbers which expreſs the length of the Sides of all Plain Triangles; that ſo you may Work by *Addition* and *Subtraction*, inſtead of *Multiplication* and *Diviſion*.

How theſe Tables are made, is not the deſign of this ſhort Tract to inform you; but how to uſe them in Calculation, muſt be well underſtood.

Suppoſe then you were to look for the Logarithm of the Side B, in *Caſe* 1. of Right-angled Triangles; whoſe Number is 768.

Look in the Table of Logarithms, under the Word *Number*, or the Letter *N*, in the firſt Column towards

the Left-hand, where are all Numbers in their Natural Order, from 1. to 1000; and right againſt 768. you will find 2. 8853612. if you uſe *Vlack*'s Tables, which I think are the beſt.

But if you uſe the Logarithms in the common Books of *Trigonometry*, the Logarithm ſought, will be leſs by one Figure, *viz.* It will be 2. 885361. Where you muſt take notice, That the firſt Figure 2, with the Point after it, is what we call the Characteriſtick, or Index of a Logarithm; becauſe it ſhews the Nature of the Abſolute correſponding Number, being always an Unite leſs, than its Number of Places.

Thus becauſe the Number 768, conſiſts of three Places, the Index of its Logarithm muſt be 2: But if it had been 7680, its Index would have

been 3, becauſe the Number hath four *Real Places*; *Real Places*, I ſay, or *Places of Integers*: For if it had been in part a Decimal Fraction, its Index would have related only to the Integer part; and will be defective or Negative, if the Number be entirely Fractional.

Thus the Index of 768.6 is as before, but 2, and the Index of 76.86 is but 1, of 7.686 is 0, and of .7686 (according to Sir *Jonas Moore*'s way, who rightly adviſes, when you have to deal with the Logarithms of Fractions, to take the Complements of the Indexes to 10) the Index of 7686 will be $\overline{9}$.

And thus may the Logarithm of any Number be readily found in the Tables, and a proper Index prefix'd to it, which muſt always be done, whether the Table give you the In-

dex or not; for if it be not there, as it is not in thoſe Tables which are in Sir *Jonas Moore*'s *Mathematical Compendium*, and ſome other Books, you muſt always ſupply it by the Rule above given.

2. To find the proper Number anſwering to any Logarithm given. As ſuppoſe the Logarithm 2.8853713 were given, to know what Number it repreſents; which is the Logarithm found by Operation, to belong to B, in *Caſe* 7. of Right-angled Triangles.

Imagine its Characteriſtick to be 3, (which is the higheſt the Tables extend to) and then look in the Tables for the Logarithm of 3.8853713; or omitting the Characteriſtick, look for 8853713. in the proper Column under the Word Logarithm: You will not find it there exactly; but that

that which comes neareſt to it, and is yet leſs than it, is 3.8853612, which anſwers to the Abſolute Number 7680.

From which Number cut off the Cypher at the End, becauſe the Index of your Logarithm was really but 2, and not 3, and the remaining three Figures 768, are to be taken for the true Number, or Length of the Side B in the Triangle; but if inſtead of 2, the Index of the Logarithm had been 1 or 0: Then you muſt have cut off one or two more Places from the Number 7680, and then the Number would have been accordingly, 76.80, or 7.680.

And if the Logarithm be not found there exactly, and that its Index ſuppoſe alſo be 4, find it firſt to four Places, as is before ſhewn.

And

And Note the common Difference under the Letter D, in the laſt Column of the Logarithms in the Book: Next take the Difference between the Logarithm propoſed, and that which is next leſs than it in the Tables; and then having Recourſe to the Tables of Proportional Parts, which are uſually Printed with the Logarithms; Seek there under the Letter D, the before found common Difference under the Column D in the Logarithms; then run your Eye along in an Horizontal Line, till you find the Number neareſt expreſſing the Difference of the Logarithm, which you found by Subtraction, and then at the Top you will have the 5th Figure to be added or annexed to the four others before found. Thus,

Suppoſe the Logarithm had been as above, 2.8853713, but with 4 inſtead

ſtead of 2 for its Index: That is, Suppoſing it to belong to a Number conſiſting of five Places.

The Number found then was 7680, againſt which, under the Letter D ſtands 565 for the common Difference, and the Difference between the Logarithm given, *viz.* 4. and the next leſs in the Tables, *viz.* 4. is 101: 8853713
And here tho' I can neither find 565, nor 101 in 8853612
the Table of Parts Proportional exactly; yet taking the neareſt to them, I find at the Top the Figure 2, which is to be annexed to 7680, and then it will be 76802; which would be the right Number anſwering to the Logarithm given, if its Index had been 4. 101

The

The Uſe of the other Tables of Sines, Tangents and Secants, is very Eaſie and Plain.

For if you would find, ſuppoſe the Logarithmick Sine of 22°. 37′. Look for 22°. on the Top of the Page, and 37′ in the firſt Column for Minutes on the Left-hand, and marked with M; and then under the Word Sine, you have 9.5849685 for the Logarithmick Sine, and 9 6197205 for the Tangent, and 10.0247521 for its Secant.

But to look for the Sine, Tangent, or Secant of any Degrees above 45, you muſt in moſt Books of Tables ſeek the Degrees at the Bottom of the Right-hand Page; and the odd Minutes muſt be reckon'd upwards alſo

alſo in the Column for Minutes on the Right-hand of that Page.

As if you were to look for the Sine, Tangent, and Secant of 67°. 23'. You will find them in the ſame Page and Line with thoſe of 22°. 37'. (whoſe Complement to 90°. is 67°. 23'.) but only you muſt account from the Bottom upwards, as is above ſaid; and the Sine will be 9.965248, the Tangent 10.380279 and the Secant 10.415031.

And as eaſie is it to find the Degrees and Minutes anſwering to any Logarithmick Sine, Tangent, or Secant, only obſerve when you cannot find it exactly, always to take the next leaſt: Thus,

If

If you were to ſeek what Degrees and Minutes anſwer to the Logarithmick Sine of 9.9652379, you will not find it exactly in the Tables, but the next Leſs is 9.9651953, which belongs to 67°. 22'.

MVSEVM BRITAN NICVM

FINIS.

BOOKS Printed for D. Midwinter *and* T. Leigh, *at the* Roſe *and* Crown *in St.* Paul's *Church-yard.*

A New and Moſt Accurate Theory of the Moon's Motion; whereby all her Irregularities may be ſolved, and her Place truly calculated to Two Minutes. Written by that *Incomparable Mathematician*, Mr. *Iſaac Newton*, and Publiſhed in *Latin*, by Mr. *David Gregory*, in his Excellent Aſtronomy.

The Merchant's Magazine: Or, Tradeſman's Treaſury: Containing, 1. Arithmetick in Whole Numbers and Fractions, Vulgar and Decimal, *&c.* 2. Merchants Accompts, *&c.* 3. Book-keeping, after a Plain, Eaſie and Natural Method, *&c.* 4. Maxims concerning Bills of Exchange, Factors and Factorage, *&c.* 5. The Port of Letters to and from Foreign Countries, *&c.* 6. An Account of the

the Commodities produc'd by all Countries, *&c.* 7. A Merchant, or Trader's Dictionary, *&c.* 8. Precedents of Merchants Writings, *&c.* wite many other things not extant before: The Fourth Edition, Corrected and Improv'd.

Comes Commercij: Or, The Trader's Companion, Containing, 1. An Exact and Useful Table, shewing the Value of any Quantity of any Commodity ready cast up, more adapted to Merchants Use than any other extant. 2. A Table calculated for Universel Use, which Use is shewn in the Solution of Questions in Multiplication, Division, Reduction, Merchandizing, and Measuring all kind of Superficies and Solids, or Gauging Vessels and Casks. 3. The Manner of Casting-up Dimensions in General, *&c.* 4. The several Customs used by Surveyors and Measurers, in measuring Glass, Wainscot, *&c.* 5. Instructions

ſtructions for Entring Goods at the Cuſtom-Houſe, *&c.* 6. Concerning Water-ſide Buſineſs, *&c.* Alſo the Charge of Wharfage, Lighterage, *&c.* 7. Practical Rules concerning Freight, Bills of Lading, Primage, and how the ſame is paid for, *&c.* 8. Rules concerning Inſuring Ships, Merchandiſe, Houſes, *&c.* with many other things never before made publick. To which is added, A Supplement concerning Simple and Compound Intereſt, *&c.* Alſo to make up Accounts of Mortgages, where the Mortgagee has receiv'd the Rent, *&c.* Both by *E. Hatton*, Gent.

A New Rational Anatomy, containing an Explanation of the Uſes of the Structure of the Body of Man, and ſome other Animals, according to the Rules of Mechanicks. By *Daniel Tauvry*, a Member of the College of Phyſicians at *Paris*. Made

Engliſh

English from the Third Edition. Illuſtrated with Sculptures.

BOOKS in the Preſs.

Lexicon Technicum Magnum: Or, An Univerſal *Engliſh* Dictionary of Arts and Sciences: Explaining not only the Terms of Art, but the Arts themſelves, *&c.* By *John Harris*, M, A. and F. R. S. in Folio.

A Treatiſe in Fluxions: Or an Introduction to Mathematical Philoſophy: Containing a full Explication of that Method, by which the moſt celebrated Geometers of the preſent Age have made ſuch vaſt Advances in Mechanical Philoſophy. A Work very uſeful for thoſe that deſire to know how to apply Mathematicks to Nature. By *Charles Hayes*, Gent. In Folio.